STABILITY OF A NONSHALLOW SPHERICAL DOME

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A version is offered of equations for large deformations of a nonshallow spherical shell analogous to the version of equations of Feodos'ev [1] for shallow shells.

Procedures are developed to overcome difficulties arising in the utilization of the method of Bubnov-Galerkin in the version of Papkovich. For the determination of the loading curve the method of transition to Cauchy's problem is used. The practical convergence of the method of Bubnov-Galerkin in this problem is examined in detail. Since the solution of the problem is determined in this case by two parameters λ and θ_0, where θ_0 is the angle of inclination of the undeformed middle surface at the fixation $\lambda = R\theta_0^2/h$, R is the radius of the middle surface, h is the thickness of the shell, the results of the analysis of the behavior of the shell are presented for various λ and θ_0 in the range $0 < \theta_0 \leqslant 0.7$, $0 \leqslant \lambda \leqslant 70$.

Tables are given for upper and lower critical pressures. Results are compared with results obtained from the theory of shallow shells and from other theories.

1. We shall examine large axisymmetrical deformations of nonshallow spherical shell loaded uniformly by a distributed external pressure. As a basis we take the following approximate relationships connecting displacements and deformations:

$$u = u_0 + \frac{z}{R}\left(u_0 - \frac{\partial w_0}{\partial \theta}\right), \qquad w = w_0, \qquad z = r - R \tag{1.1}$$

$$\varepsilon_{rr} = \frac{\partial w_0}{\partial r} = 0, \qquad \varepsilon_{\varphi\varphi} = \varepsilon_{\varphi\varphi}^{(0)} + z\varepsilon_{\varphi\varphi}^{(1)}, \qquad \varepsilon_{\theta\theta} = \varepsilon_{\theta\theta}^{(0)} + z\varepsilon_{\theta\theta}^{(1)} \tag{1.2}$$

$$\varepsilon_{\varphi\varphi}^{(0)} = \frac{1}{R\sin\theta}(w_0 \sin\theta + u_0\cos\theta), \qquad \varepsilon_{\varphi\varphi}^{(1)} = -\frac{\operatorname{ctg}\theta}{R^2}\frac{\partial w_0}{\partial\theta} \tag{1.3}$$

$$\varepsilon_{\theta\theta}^{(0)} = \frac{1}{R}\left(\frac{\partial u_0}{\partial\theta} + w_0\right) + \frac{1}{2R^2}\left(\frac{\partial w_0}{\partial\theta}\right)^2, \qquad \varepsilon_{\theta\theta}^{(1)} = -\frac{1}{R^2}\cdot\frac{\partial^2 w_0}{\partial\theta^2} \tag{1.4}$$

Here u_0 and w_0 are tangential and normal displacements of points of the middle surface, R is the radius of the middle surface, r is the moving radius, θ is the polar angle (Fig. 1).

The relationship between the components of deformation and stresses are written in the following form:

$$T_1 = E_1(\varepsilon_{\theta\theta}^{(0)} + \mu\varepsilon_{\theta\theta}^{(1)}), \qquad T_2 = E_1(\varepsilon_{\theta\theta}^{(1)} + \mu\varepsilon_{\theta\theta}^{(0)}) \qquad (E_1 = Eh/(1-\mu^2)) \tag{1.5}$$

$$M_1 = E_2(\varepsilon_{\varphi\varphi}^{(0)} + \mu\varepsilon_{\varphi\varphi}^{(1)}), \qquad M_2 = E_2(\varepsilon_{\varphi\varphi}^{(1)} + \mu\varepsilon_{\varphi\varphi}^{(0)}) \qquad (E_2 = Eh^3/12(1-\mu^2)) \tag{1.6}$$

Here T_1, T_2 are the shear resultants, M_1, M_2 are bending moments, E_1 is the rigidity of the shell in tension, E_2 is the rigidity of the shell in bending.

Eqs. (1.5) and (1.6) are obtained on the basis of Hooke's law.

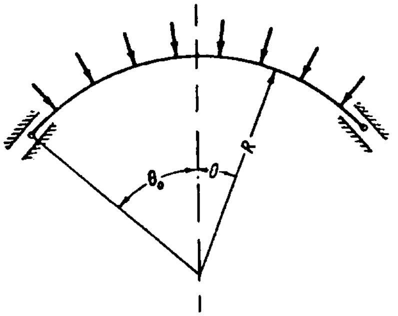

Fig. 1

Then from Lagrange's principle the following system of equations arises for the equilibrium of the shell:

$$\frac{\partial T_1}{\partial \theta} + (T_1 - T_2)\operatorname{ctg}\theta = 0 \qquad (1.7)$$

$$\frac{\partial^2 M_1}{\partial \theta^2} + \left(\frac{\partial M_1}{\partial \theta} - \frac{\partial M_2}{\partial \theta}\right)\operatorname{ctg}\theta - (M_1 - M_2) -$$

$$- R(T_1 + T_2) - R^2 T_1 \varepsilon_{\varphi\varphi}^{(1)} - \omega R T_2 \operatorname{ctg}\theta - qR^2 = 0 \qquad (1.8)$$

Here ω is the angle of rotation of the normal with respect to the middle surface $r = R$, which is given by the relationship

$$\omega = \frac{1}{r}\frac{\partial w}{\partial \theta} - \frac{u}{r} \qquad (1.9)$$

In the derivation of Eqs. (1.7) and (1.8) identification of the internal geometry of the shell with the geometry in the plane was not made. This distinguishes this system from known versions of equations of equilibrium in the theory of shallow shells.

We shall examine a sliding hinge fixation which has the following boundary conditions: for $\theta = \theta_0$, where θ_0 is the angle characterizing the fixation location of the dome (Fig. 1)

$$\frac{d^2 w}{d\theta^2} + \mu \operatorname{ctg}\theta \frac{dw}{d\theta} = 0, \quad w = 0, \quad \psi = 0 \qquad (1.10)$$

Here ψ is a stress function

$$T_1 = \frac{\psi}{\sin\theta}, \qquad T_2 = \frac{1}{\cos\theta}\frac{d\psi}{d\theta} \qquad (1.11)$$

If one now substitutes (1.3), (1.4), (1.6) and (1.11) into (1.8), then takes $dw/d\theta = \varphi$ and integrates the newly obtained equation with respect to θ between the limits from 0 to θ, we will finally have

$$\frac{E_2}{R^2}\left[\frac{d^2\varphi}{d\theta^2} + \operatorname{ctg}\theta\frac{d\varphi}{d\theta} - \varphi(\mu + \operatorname{ctg}^2\theta)\right] - \frac{\varphi\psi}{\sin\theta} +$$

$$+ \frac{R}{\sin\theta}\int_0^\theta \frac{d/d\theta\,(\psi\sin\theta)}{\cos\theta}\,d\theta + 2qR^2\sin^2\frac{\theta}{2}\frac{1}{\sin\theta} = 0 \qquad (1.12)$$

Two unknown functions w and ψ enter into Eq. (1.12). The second relationship connecting w and ψ will be the equation of compatibility. In order to obtain it we substitute (1.1) to (1.4) into (1.11) and eliminate u. As a result we shall have

$$\frac{d^2\psi}{d\theta^2} + \frac{d\psi}{d\theta}(\operatorname{ctg}\theta + 2\operatorname{tg}\theta) - \psi(\operatorname{ctg}^2\theta + \mu) +$$

$$+ \frac{E_1(1-\mu^2)}{R}\left[\frac{\cos^2\theta}{2R\sin\theta}\left(\frac{dw}{d\theta}\right)^2 - \cos\theta\frac{dw}{d\theta} - w\sin\theta\right] = 0 \qquad (1.13)$$

Let us introduce nondimensional quantities with the aid of the following relationships

$$\psi = -\frac{Eh^3}{R^2\varepsilon}\psi_0, \quad \varphi = -\frac{h}{\varepsilon}\varphi_0, \quad w = -hw_0, \quad \lambda = \frac{R\varepsilon^2}{h}$$

$$\theta = \rho\varepsilon, \quad \varepsilon = \theta_0, \quad w_0 = \int\limits_1^\rho \varphi_0(t)\,dt, \quad q_0 = \frac{q}{E}\left(\frac{R\varepsilon}{h}\right)^4$$

Eqs. (1.12) and (1.13) and conditions (1.10) are represented in the following form:

$$\frac{d^2\psi_0}{d\theta^2}\frac{\sin\varepsilon\rho}{\varepsilon} + \frac{d\psi_0}{d\theta}\left(\cos\varepsilon\rho + 2\frac{\sin^2\varepsilon\rho}{\cos\varepsilon\rho}\right) - \psi_0\left(\mu\varepsilon\sin\varepsilon\rho + \frac{\varepsilon\cos^2\varepsilon\rho}{\sin\varepsilon\rho}\right) =$$

$$= \varphi_0\frac{\sin 2\varepsilon\rho}{2\varepsilon}\lambda + \lambda\sin^2\varepsilon\rho w_0 + \frac{\varphi_0^2\cos^2\varepsilon\rho}{2} \qquad (1.14)$$

$$\frac{1}{1-\mu^2}\left[\frac{d^2\varphi_0}{d\theta^2}\frac{\sin\varepsilon\rho}{\varepsilon} + \frac{d\varphi_0}{d\theta}\cos\varepsilon\rho - \varphi_0\left(\mu\varepsilon\sin\varepsilon\rho + \frac{\varepsilon\cos^2\varepsilon\rho}{\sin\varepsilon\rho}\right)\right] =$$

$$= -12\left[\varphi_0\psi_0 + \lambda\int\limits_0^\rho \frac{d/dt\,(\psi_0(\sin\varepsilon t)/\varepsilon)}{\cos\varepsilon t}\,dt\right] + 6q_0\rho^2\left(\frac{2\sin\varepsilon\rho/2}{\varepsilon\rho}\right)^2 \qquad (1.15)$$

$$\left[\frac{d\varphi_0}{d\theta} + \mu\,\varepsilon\varphi_0\operatorname{ctg}\varepsilon\rho\right]_{\rho=1} = 0, \qquad w_0\,|_{\rho=1} = 0, \qquad \psi_0\,|_{\rho=1} = 0 \qquad (1.16)$$

2. Let us assume that it is necessary to determine the loading curve for the dome, i.e. to find the dependence of q_0 on its own nondimensional displacement at the center $w_0|_{\rho=0} = f$. It is easy to see that f is determined by the relationship:

$$f = \int\limits_1^0 \varphi_0(t)\,dt \qquad (2.1)$$

System (1.14), (1.15) will be solved by the method of Bubnov-Galerkin. Let us assume

$$\varphi_0 = \sum_{i=0}^{N} C_{i+1}(\rho^{2i+3} - \gamma_i\rho^{2i+1}) \quad \left(\gamma_i = \frac{2i+3+\mu\varepsilon\operatorname{ctg}\varepsilon}{2i+1+\mu\varepsilon\operatorname{ctg}\varepsilon}\right) \qquad (2.2)$$

Here, boundary conditions (1.16) are satisfied.

It is necessary to note that the application of the procedure by Papkovich is complicated in this case in contrast to the case of shallow shells, since Eq. (1.14) is an equation with variable coefficients. If one takes into account relationships (2.2), the following expressions are obtained for φ_0 and w_0:

$$\varphi_0 = \sum_{i=0}^{N} A_i\rho^{2i+1} \qquad (A_i = C_i - \gamma_i C_{i+1},\ C_0 = 0) \qquad (2.3)$$

$$w_0 = \sum_{i=0}^{N}\frac{A_i}{2i+2}\rho^{2i+2} - w_1 \qquad \left(w_1 = \sum_{i=0}^{N} C_{i+1}\left(\frac{1}{2i+4} - \frac{\gamma_i}{2i+2}\right)\right) \qquad (2.4)$$

Therefore the right-hand part of (1.14) is an entire function of ρ.

$$\frac{d^2\psi_0}{d\theta^2}\frac{\sin\varepsilon\rho}{\varepsilon} + \frac{d\psi_0}{d\theta}\left(\cos\varepsilon\rho + 2\frac{\sin^2\varepsilon\rho}{\cos\varepsilon\rho}\right) - \psi_0\left(\mu\varepsilon\sin\varepsilon\rho + \frac{\varepsilon\cos^2\varepsilon\rho}{\sin\varepsilon\rho}\right) = \sum_{n=0}^{\infty} f_n\rho^{2n+2} \qquad (2.5)$$

It is natural to look for a solution of (2.5) in the form

$$\psi_0 = \sum_{n=0}^{\infty} f_n \psi_n(\rho) \qquad (\varphi_n = \varphi_n^* - \delta_n \varphi_n^{**}) \tag{2.6}$$

$$\psi_n^* = \rho^{2n+3} \sum_{k=0}^{\infty} d_k^{(n)} \rho^{2k}, \qquad \psi_n^{**} = \rho \sum_{k=0}^{\infty} e_k \rho^{2k} \tag{2.7}$$

Here ψ_n^* is the particular solution of the inhomogeneous Eq. (2.5) and ψ_n^{**} is the general solution of the corresponding homogeneous solution. Arbitrary constants

$$\delta_n = \Big(\sum_{k=0}^{\infty} d_k^{(n)}\Big)\Big(\sum_{k=0}^{\infty} e_k\Big)^{-1} \tag{2.8}$$

are determined from the condition $\psi_0 |_{\rho=1} = 0$.

Substituting (2.6) to (2.8) into (2.5) and equating coefficients of the left and right sides of the transfomed equation for equal powers of ρ, we obtain

$$d_0^{(n)} = \frac{1}{4(n+1)(n+2)} \tag{2.9}$$

$$d_k^{(n)} = -\frac{1}{4(n+k+1)(n+k+2)} \times \tag{2.10}$$

$$\times \Big\{ \sum_{s=0}^{k-1} d_s^{(n)} \frac{(\varepsilon^2)^{k-s}}{(2k-2s)!} [(2n+2s+3)\, 2\, |E_{2k-2s}| - (4^{k-s}-2)\, |B_{2k-2s}|] -$$

$$- \sum_{s=0}^{k-1} d_s^{(n)} \frac{(-\varepsilon^2)^{k-s}}{(2k-2s-1)!} \Big[(1-\mu) + \frac{n+s+1.5}{k-s} - \frac{(n+s+1.5)(n+s+1)}{(k-s)(k-s+0.5)}\Big]\Big\}, \quad k \geqslant 1$$

$$e_0 = 1, \qquad e_k = d_k^{(1)}, \quad k \geqslant 1 \tag{2.11}$$

Here E_n are Euler's numbers, B_n are Bernoulli's numbers.

Relationships (2.9) to (2.11) are correct when $\theta < \frac{1}{2}\pi$. We substitute (2.2) and (2.6) into the left part of (1.15) and require that the obtained expression be orthogonal $(\rho^{2r+3} - \gamma_r \rho^{2r+1})$, $r = 1, 2, \ldots, N$.

In this manner we obtain an algebraic system of equations of the third order for dermination of C_{i+1}:

$$\sum_{i=0}^{N} C_{i+1} [A_{im}^{(1)} + \lambda^2 A_{im}^{(2)}] + \lambda \sum_{i=0}^{N} \sum_{n=0}^{N} C_{i+1} C_{n+1} A_{imn} +$$

$$+ \sum_{i=0}^{N} \sum_{n=0}^{N} \sum_{j=0}^{N} C_{i+1} C_{n+1} C_{j+1} A_{injm} = A_m q_0 \tag{2.12}$$

Coefficients $A_{im}^{(1)}, A_{im}^{(2)}, A_{imn}, A_{injm}$ depend on the parameter $\varepsilon = \theta_0$. Consequently the nonlinear problem of stability under examination in this case will have two parameters ($\lambda = R\theta_0^2 / h$ and $\varepsilon = \theta_0$), which substantially complicates the examination of the problem.

For determination of C_{n+1} and q_0 we utilize the idea presented in [2] applied to the

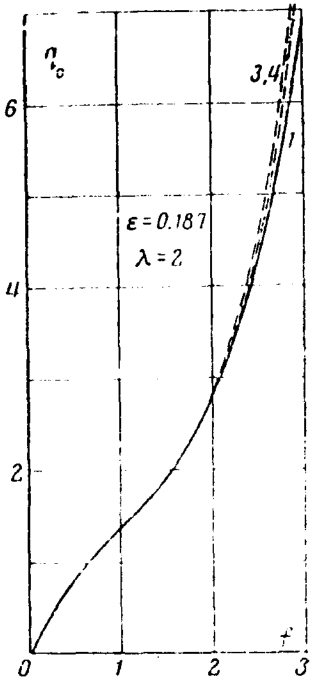

Fig. 2

investigation of a shallow spherical dome . As independent parameter either the nondimensional displacement

$$f = \sum_{k=0}^{\infty} C_{k+1}\left(\frac{\gamma_k}{2k+2} - \frac{1}{2k+4}\right) \tag{2.13}$$

or the nondimensional pressure q_0 can be taken. Correspondingly, from (2.12) and (2.13) we obtain the following two systems of differential Eqs:

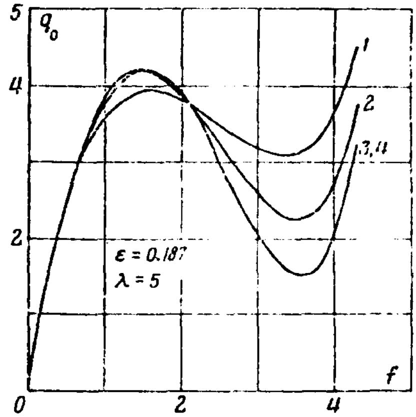

Fig. 3

$$\sum_{i=0}^{N} \frac{dC_{i+1}}{df}\left[(A_{im}^{(1)} + \lambda^2 A_{im}^{(2)}) + \lambda \sum_{n=0}^{N}(A_{inm} + A_{nim})C_{n+1} + \right.$$

$$\left. + \sum_{n=0}^{N}\sum_{j=0}^{N}(A_{injm} + A_{nijm} + A_{jinm})C_{n+1}C_{j+1}\right] - \frac{dq_0}{df}A_m = 0 \tag{2.14}$$

$$\sum_{i=0}^{N}\left(\frac{\gamma_i}{2i+2} - \frac{1}{2i+4}\right)\frac{dC_{i+1}}{df} = 1 \quad (m = 1, 2 \ldots, N) \tag{2.15}$$

$$\sum_{i=1}^{N} \frac{dC_{i+1}}{dq_0}\left[(A_{im}^{(1)} + \lambda^2 A_{im}^{(2)}) + \lambda \sum_{n=0}^{N}(A_{inm} + A_{nim})C_{n+1} + \right.$$

$$\left. + \sum_{n=0}^{N}\sum_{j=0}^{N}(A_{injm} + A_{nijm} + A_{jinm})C_{n+1}C_{j+1}\right] - A_m = 0 \tag{2.16}$$

$$\sum_{i=0}^{N}\left(\frac{\gamma_i}{2i+2} - \frac{1}{2i+4}\right)\frac{dC_{i+1}}{dq_0} = \frac{df}{dq_0} \qquad (m = 1, 2, \ldots, N) \tag{2.17}$$

As initial data we may take elements of the unstressed state of the shell in the absence of loading, i.e. for $f = 0$, $q_0 = 0$, $C_{i+1} = 0$. The integration of systems (2.14), (2.15) and (2.16), (2.17) was carried out by the Runge-Kutta method. In this case it is convenient to integrate system (2.14), (2.15) as long as dq_0/df is not very large.

In the opposite case it is appropriate to integrate system (2.16), (2.17).

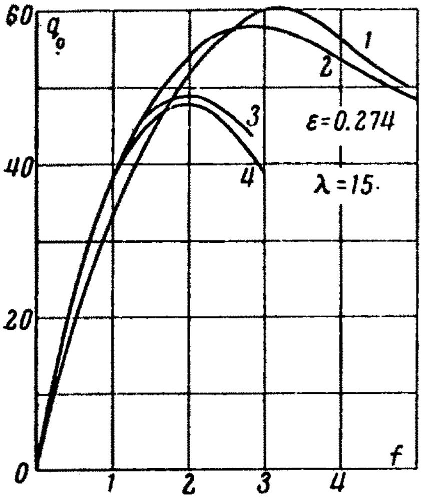

Fig. 4

The program was composed for the electronic digital computer 'Minsk-12'. It consisted of standard blocks and allowed automatic switching of integration from one system to the other.

3. Let us examine results of computations. Curves $q_0 - f$ were computed for the following

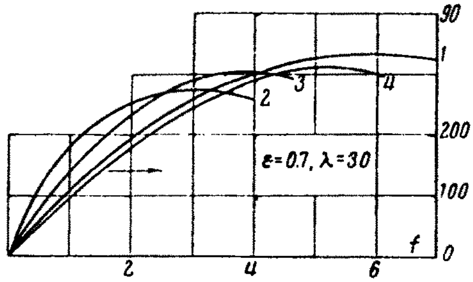

Fig. 5

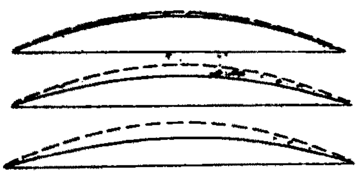

Fig. 6

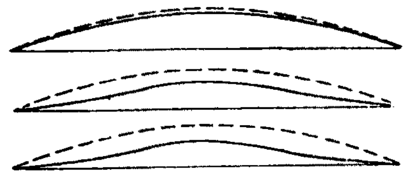

Fig. 7

combinations of values of parameters λ and ε:

$$\varepsilon = 0.187\ \{\lambda = 2, 4, 5, 12, 15, 30, 50, 70\}$$
$$\varepsilon = 0.273\ \{\lambda = 5, 12, 15, 30, 50, 70\}$$
$$\varepsilon = 0.5\ \{\lambda = 15, 30, 50, 70\}$$
$$\varepsilon = 0.7\ \{\lambda = 30\}$$

The selection of values of parameter λ in its dependence on ε was determined from the condition

$$R / h \geqslant 50 \tag{3.1}$$

It was found that the method of Bubnov-Galerkin gives satisfactory accuracy on the basis of the fourth approximation for the upper and also the lower critical loadings in the case $\lambda \leqslant 5$, $\varepsilon \leqslant 0.3$. In the other cases, reliable results are obtained only for upper critical loadings. Table 1 is presented for characterization of the rate of convergence of values q_0.

It is evident from the Table that in the case $\lambda \leqslant 5$, $\varepsilon \leqslant 0.3$ the fourth approximation differs from the third by no more than 0.2%. In the

Table 1

f	1	2	3	4
	ε=0.187, λ=2			
0.2	0.3885	0.3990	0.3997	0.3994
1.0	1.426	1.418	1.416	1.416
2.0	2.902	2.922	2.928	2.928
3.0	6.065	7.371	7.512	7.532
4.0	14.95	18.29	18.64	18.94

(Table 1, continued on the next page)

(Table 1 continued from previous page)

f	1	2	3	4
	$\varepsilon=0.187$,	$\lambda=4$		
0.2	0.7739	0.8294	0.8364	0.8331
1.0	2.479	2.591	2.600	2.600
1.6	2.797	2.818	2.811	2.814
2.0	2.821	2.731	2.695	2.698
2.6	2.932	2.655	2.550	2.548
3.0	3.265	2.920	2.793	2.785
	$\varepsilon=0.187$,	$\lambda=5$		
0.2	1.074	1.168	1.183	1.174
1.0	3.541	3.795	3.822	3.820
1.6	3.973	4.161	4.176	4.184
3.0	3.178	2.585	2.150	2.160
3.6	3.210	2.264	1.534	1.531
4.0	3.722	2.764	2.129	2.103
	$\varepsilon=0.187$,	$\lambda=12$		
0.2	4.479	5.999	6.111	5.788
1.0	20.57	23.57	23.63	23.16
2.0	30.96	34.04	30.23	30.80
2.6	32.87	36.13	28.12	28.34
	$\varepsilon=0.187$,	$\lambda=70$		
0.2	174.0	235.6	104.4	113.1
1.0	839.8	1020	490.8	523.6
2.0	1605	1678	910.4	951.2
3.0	2299	2053	1268	1298
4.0	2922	2213	1570	1581
	$\varepsilon=0.273$,	$\lambda=5$		
0.2	1.055	1.149	1.164	1.160
1.0	3.482	3.736	3.764	3.766
1.6	3.912	4.100	4.115	4.125
2.0	3.803	3.854	3.830	3.844
	$\varepsilon=0.273$,	$\lambda=12$		
0.2	5.068	5.934	6.051	6.013
1.0	20.54	23.37	23.52	23.37
2.0	30.42	33.86	30.40	30.60
	$\varepsilon=0.273$,	$\lambda=70$		
0.2	170.2	240.3	102.5	114.4
1.0	821.4	1037	482.6	544.6
2.0	1570	1702	897.2	1016
3.0	2248	2080	1254	1413
4.0	2857	2245	1558	1729
5.0	3400	2252	1812	1955
6.0	3879	2137	2010	2081
	$\varepsilon=0.5$,	$\lambda=15$		
0.2	7.085	9.085	9.037	8.050
1.0	29.86	36.65	37.08	34.35
2.2	49.12	55.79	51.22	52.59

(Table 1 continued on the next page)

case of large λ this difference does not exceed 3%.

In Fis. 2 and 3 the dependence of q_0 on f is shown, obtained in the first to fourth approximations when $\varepsilon = 0.187$; $\lambda = 2.5$. From these graphs it is evident that the third and fourth approximations are practically indistinguishable. In cases $\varepsilon = 0.273$; $\lambda = 15$ and $\varepsilon = 0.7$; $\lambda = 30$ satisfactory agreement between the third and the fourth approximation is achieved only on segments of loading curves shown in Figs. 5, 4 and 5. A summary table of upper critical loadings is presented for various values of ε and λ (Table 2). It may be noted that with increasing λ for a given ε the upper critical values q_0^+ increase as is evident from the presented table.

In the determination of displacements it was possible to obtain satisfactory accuracy on the basis of the fourth approximation. In this connection the approximation of the deflection curve was carried out by means of a polynomial of tenth degree in accordance with (2.2).

In Figs. 6 to 8 various stages of loading of shells are represented for several values of ε and λ. In Fig. 6 the case $\varepsilon = 0.273$; $\lambda = 5$ is examined.

Position I corresponds to loading q_0, which is less than the upper critical value. Position II corresponds to loading q_0^+ which is the upper critical loading. The third position corresponds to loading q_0 exceeding the upper critical value.

In Fig. 7 the development of equilibrium forms of the shell is given for values of $\varepsilon = 0.273$; $\lambda = 12$, and, finally, in Fig. 8 three positions of nonshallow spherical segment are depicted for $\varepsilon = 0.7$ and $\lambda = 30$.

The magnitudes of upper critical values obtained in this paper in the case of $\varepsilon \leq 0.2$; $\lambda \leq 5$ differ little from quantities q_0^+ for shallow spherical shells.

For comparison we note that the value q_0^+ calculated in the fourth approximation from the theory of shallow domes will be $q_0^+ = 2.84$

(Table 1 continued from previous page)

f	1	2	3	4
		$\varepsilon=0.5$, $\lambda=70$		
0.2	153.5	268.0	93.51	100.9
1.0	740.3	1124	443.1	470.8
2.0	1414	1813	831.4	865.6
4.0	2569	2389	1483	1487
5.0	3055	2420	1760	1746
		$\varepsilon=0.7$, $\lambda=30$		
0.2	23.90	46.04	28.50	21.25
1.0	109.5	176.3	132.6	97.25
2.0	195.1	253.3	232.0	175.8
3.0	258.7	273.2	288.9	241.4
4.0	302.1	259.0	303.0	291.5

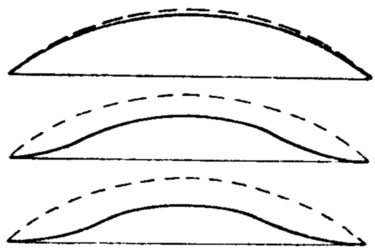

Fig. 8

($\lambda = 4$), $q_e^+ = 4.22$ ($\lambda = 5$) and the corresponding values from theory of nonshallow domes are $q_e^+ = 2.84$, $q_e^+ = 4.22$.

It is appropriate to mention the noticeable effect of the nonshallow character of the shell on the lower critical values. Thus for $\lambda = 4$ we have $q_e^- = 2.78$ according to the theory of shallow shells and $q_e = 2.53$ from the theory of nonshallow shells. For $\lambda = 5$, $q_e^- = 3.00$ and $q_e^- = 1.48$, respectively. It may be noted that the theory of A.V. Pogorelov gives substially higher values for upper critical loadings

Table 2

ε	$\lambda=70$	50	30	15	12	5	4
0.187	2121	901.1	251.3	50.05	30.8	4.184	2.814
0.273	2105	909.8	245.7	48.36	30.4	4.125	
0.5	2441	1026	276.5	52.85			
0.7			314.5				

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